

Stability of a Random Multiple Access Channel with “Success-Failure” Feedback

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We consider a model of a decentralized multiple access system with a non-standard binary feedback where the empty and collision situations cannot be distinguished. We show that, like in the case of a ternary feedback, for any input rate $\lambda < e^{-1}$, there exists a “doubly randomized” adaptive transmission protocol which stabilizes the behavior of the system. We discuss also a number of related problems and formulate some hypotheses.

Keywords: random multiple access; binary feedback; positive recurrence; (in)stability; Foster criterion.

1 Introduction

We consider a decentralised multiple access system model with an infinite number of users, a single transmission channel, and an adaptive transmission protocol that does not use the individual history of messages. With any such a protocol, all users transmit their messages in time slot $(n, n+1)$ with equal probabilities p_n that depend on the history of feedback from the transmission channel.

Algorithms with ternary feedback “Empty-Success-Collision” were introduced in [13] and [2]. It is assumed that the users can observe the channel output and distinguish among three possible situations: either no transmission (“Empty”) or transmission from a single server (“Success”) or a collision of messages from two or more users (“Conflict”). It is known since the 80’s (see e.g. [5], [9]) that if the feedback is ternary, then the channel capacity is e^{-1} : if the input rate is below e^{-1} , then there is a stable transmission protocol; and if the input rate is above e^{-1} , then any transmission protocol is unstable. A stable protocol may be constructed recursively as follows: given probability p_n in time slot $(n, n+1)$ and a feedback at time $n+1$, probability p_{n+1} is bigger than p_n if the slot $(n, n+1)$ was empty, $p_{n+1} = p_n$ if there was a successful transmission, and p_{n+1}

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is smaller than p_n if there is a conflict. Hajek [5] considered a multiplicative increase/decrease, and Mihajlov [9] an additive one.

It is also known, see e.g. [7] and [10] that similar results (existence/nonexistence of a stable protocol if the input rate is below/above e^{-1}) hold for systems with either “Empty–Nonempty” binary feedback or “Conflict–Nonconflict” feedback.

In this paper we show that the channel capacity is again e^{-1} for a third type of the binary feedback, “success–failure” feedback, which might also be called “success–nonsuccess” feedback. The problem is interesting from a practical point of view, because in order for a receiver to distinguish between “collision” and “no transmission”, it would have to differentiate between the increased energy or structure present when there is a collision of two or more packets, from thermal noise. That can be difficult or impossible for some receivers to do.

Tsybakov and Beloyarov, [11] and [12], introduced and studied a model with the “success–failure” feedback, but with an extra option. There is a selected in advance station which works as follows. Given a nonsuccess feedback, the station may send, in the next time slot, a testing package to recognize what has happened, either empty slot or collision. Clearly, if an algorithm uses this option regularly, it is an algorithm with the ternary feedback which works slower than the conventional one (uses two time slots instead of one in the case of nonsuccess). In [11] and [12], the authors introduce a class of algorithms that send a testing package from time to time only and show (numerically) that the lower is the rate of using this option, the closer to e^{-1} is the throughput.

In this paper, we do not allow testing of packages. Our approach to the problem is to introduce a further (second) randomization. We consider a new class of “doubly randomised” protocols and show that, for any pair of numbers $0 < \lambda_1 < \lambda_0 < e^{-1}$, there exists a protocol from the class that makes stable a system with input rate λ , for any $\lambda \in [\lambda_1, \lambda_0]$. Then we formulate two conjectures on stability of other classes of protocols and, in particular of protocols that do not depend on an actual value of λ . Our stability result is based on a generalized Foster criterion and the fluid approximation approach, see e.g. [4].

In a recent paper [15] (see also [6]), a stability result has been obtained for a similar model with “success–failure” feedback where a user may also take into account the arrival time of its message. It was shown that if the input rate is below 0,317, then there exist stable algorithms (called “algorithms with delayed intervals”). The results of our paper are stronger in two directions: we show that a stable protocol exists if the input rate is below e^{-1} (clearly, $e^{-1} > 0.317$) and that there is no need to use the information about arrival times.

There is an interesting question which seems to be open: assume we know arrival times of messages. Could this extra information increase an actual channel capacity?

The paper is organised as follows. Section 2 contains the description of the model and of the class of transmission protocols under consideration, as well as the statement of the main result. Its proof is presented in Section 3. Then in Section 4 we introduce two more classes of protocols and formulate conjectures on their stability.

2 The Model and the Class of Protocols

We consider (a variant of) a multi-access system introduced in [14]. There is an infinite number of users and a single transmission channel available to all of them. Users exchange their messages

using the channel. Time is slotted and all message lengths are assumed to be equal to the slot length (and equal to one).

The input process of messages $\{\xi_n\}$ is assumed to be i.i.d. with finite mean $\lambda = \mathbf{E}\xi_1$, here ξ_n is a total number of messages arriving within time slot $[n, n+1)$ (we call it “time slot n ”, for short).

The systems operates according to an “adaptive ALOHA protocol” that may be described as follows. There is no coordination between the users, and at the beginning of time slot n each message present in the system is sent to the channel with probability p_n , independently of everything else. So given that the total number of messages is N_n , the number of those sent to the channel, B_n , has conditionally the Binomial distribution $B(N_n, p_n)$ (here $B_n \equiv 0$ if $N_n = 0$). Let $J_n = 1$ if $B_n = 1$ and $J_n = 0$, otherwise. If $J_n = 1$, then there is a successful transmission within time slot n . Otherwise there is either an empty slot ($B_n = 0$) or a collision of messages ($B_n \geq 2$), so there is no transmission. Then the following recursion holds:

$$N_{n+1} = N_n - J_n + \xi_{n+1}. \quad (1)$$

A transmission protocol is determined by sequence $\{p_n\}$. We consider “decentralised” protocols: the numbers N_n , $n = 1, 2, \dots$ are not observable, and only values of past J_k , $k < n$ are known. We consider protocols where $\{p_n\}$ are defined recursively in the Markovian fashion: p_n is a (random) number that depends on the history of the system only through p_{n-1} and J_{n-1} . Then a 2-dimensional sequence (N_n, p_n) , $n = 1, 2, \dots$ forms a time-homogeneous Markov chain.

In the paper, we introduce three classes of decentralised protocols, prove a stability theorem for the first class and conjecture similar results for the two others. To describe these protocols, we introduce additional notation.

Let $N_1 \geq 0$ be the initial number of messages in the system and $S_1 \geq 1$ a positive number (which is an “estimator” of unknown S_1). Let further $\beta \in (0, 1)$, $C > 0$ and $D > 0$ be three positive parameters, and let $\{I_n\}$ be an i.i.d. sequence that does not depend on the previous r.v.’s, with $\mathbf{P}(I_n = 1) = 1 - \mathbf{P}(I_n = 0) = 1/2$.

Remark 1. In what follows, one can assume a sequence of estimators $\{S_n\}$ to take integer values only, by assuming that D and C are integer-valued. But this is not needed, in general.

The class \mathcal{A}_1 of algorithms is determined by β , $C > 0$, $D > 0$, $\{J_n\}$ and $\{I_n\}$ as follows. The transmission probabilities p_n and the numbers S_n are updated recursively: given S_n , we let

$$p_n = \begin{cases} \beta/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1, \end{cases}$$

and then

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + CD & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - CD, 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

We denote such an algorithm by $A_1(C, D, \beta) \in \mathcal{A}_1$.

The two other classes are defined in Section 4.

We can see that, with any algorithm introduced above, sequence $\{(N_n, S_n)\}$ forms a time-homogeneous Markov chain.

Definition 1. We say that a Markov chain $\{(N_n, S_n)\}$ is *positive recurrent* if there exists a compact set $\mathcal{K} \in \mathcal{R}_+^2$ such that

- for any pair of initial values $(N_1, S_1) = (N, S)$,

$$\tau_{N,S} = \min\{n \geq 1 : (N_n, S_n) \in \mathcal{K}\} < \infty \quad \text{a.s.}$$

- further,

$$\sup_{(N,S) \in \mathcal{K}} \mathbf{E} \tau_{N,S} < \infty.$$

A Markov chain $\{(N_n, S_n)\}$ is *Harris-ergodic* if there is a probability distribution μ such that, for any initial value $(N_1, S_1) = (N, S)$, the distributions of (N_n, S_n) converge to μ in the total variation,

$$\sup_A |\mathbf{P}((N_n, S_n) \in A) - \mu(A)| \rightarrow 0, \quad n \rightarrow \infty \quad (2)$$

where the supremum is taken over all Borel sets A in \mathcal{R}_+^2 .

It is well-known (see e.g. [8]) that a positive recurrent Markov chain $\{(N_n, S_n)\}$ is Harris-ergodic if it is aperiodic and there exist a positive integer m , a probability measure φ , and a positive number c such that

$$\mathbf{P}((N_m, S_m) \in \cdot \mid (N_1, S_1) = (N, S)) \geq c\varphi(\cdot), \quad (3)$$

for all $(N, S) \in \mathcal{K}$.

Definition 2. We say that Markov chain $\{(N_n, S_n)\}$ is *transient* if there is an initial value $(N_1, S_1) = (N, S)$ such that $N_n + S_n \rightarrow \infty$ a.s., as $n \rightarrow \infty$.

Definition 3. Algorithm A is *stable* if the underlying Markov chain (N_n, S_n) determined by A is Harris-ergodic, and *unstable* if the underlying Markov chain is transient.

Here is our main result.

Theorem 1. Let $0 < \lambda_1 < \lambda_0 < e^{-1}$ be any two numbers. There exist $C > 0$, $\beta_1 \in (0, 1)$ such that, for any fixed $\beta \in (\beta_1, 1)$, there exists $D_0 = D_0(\lambda_0, \lambda_1, \beta)$ such that, for any $D \geq D_0$, algorithm $A_1(C, D, \beta)$ is stable for any input rate $\lambda \in (\lambda_0, \lambda_1)$.

3 Proof of Theorem 1

We need to introduce a number of auxiliary functions: for positive numbers β, λ, C, D and for $0 \leq z < \infty$, let

$$j_1(z, \beta) = \frac{\beta z}{2} e^{-\beta z}, \quad j_2(z) = \frac{z}{2} e^{-z}; \quad (4)$$

$$j(z, \beta) = j_1(z, \beta) + j_2(z); \quad (5)$$

$$a(z, \beta) = \lambda - j(z, \beta); \quad (6)$$

$$b(z, \beta) = C(1 - j(z, \beta)) + CD(j_1(z, \beta) - j_2(z)); \quad (7)$$

$$r(z, \beta) = a(z, \beta) - zb(z, \beta). \quad (8)$$

Clearly, for any $\beta > 0$ and as $z \rightarrow \infty$, $j_1(z, \beta)$, $j_2(z)$ and $j(z, \beta)$ tend to 0, $a(z, \beta) \rightarrow \lambda$, $b(z, \beta) \rightarrow C$, and $r(z, \beta) \rightarrow -\infty$.

We rely on the following auxiliary result.

Lemma 1. *The functions $j(z), a(z), b(z)$ and $r(z)$ satisfy the following conditions:
for any $0 < \lambda_0 < \lambda_1 < e^{-1}$, there exists $\beta_1 \in (0, 1)$ such that, for any $\lambda \in (\lambda_0, \lambda_1)$, $\beta \in (\beta_1, 1)$
and for any $C \geq C_1 := \frac{\lambda_1 + 1}{1 - e^{-1}}$ and $D \geq D(C)$ (where $D(C)$ is specified in the proof),*

- *equation $a(z) = 0$ has two roots $0 < z_1 < z_2 < \infty$;*
- *equation $b(z) = 0$ has two roots $0 < t_1 < t_2 < \infty$;*
- *$0 < t_1 < z_1 < t_2 < z_2$;*
- *$r(z) > 0$ for $z \in (0, t_1)$ and $r(z) < 0$ for $z > z_2$ and, therefore, all roots to equation $r(z) = 0$ lie in the interval (z_1, t_2) .*

PROOF. Introduce a further function

$$b_1(z, \beta) = 1 - j(z, \beta) + D(j_1(z, \beta) - j_2(z, \beta)).$$

Then $b(z, \beta) = Cb_1(z, \beta)$.

We know that ze^{-z} (and also $j_2(z)$) is increasing in z if $z \in (0, 1)$ and decreasing if $z > 1$. Then, for $\beta < 1$, $j_1(z, \beta)$ is increasing if $z < 1/\beta$ and decreasing if $z > 1/\beta$. Further, $m(\beta) := \min_{1 \leq z \leq 1/\beta} j(z, \beta)$ is strictly positive and tend to 1 as $\beta \uparrow 1$.

For any $\lambda_1 \in (0, e^{-1})$ and any $\varepsilon \in (0, e^{-1} - \lambda_1)$, one can choose $\widehat{\beta}_1 < 1$ so close to 1 that $m(\beta) \geq \lambda_1 + \varepsilon$, for all $\beta \in [\widehat{\beta}_1, 1)$. Then for any $\lambda \in (0, \lambda_1]$ and any $\beta \in [\widehat{\beta}_1, 1)$, equation $j(z, \beta) = \lambda$ has two roots, $z_1(\beta, \lambda)$ and $z_2(\beta, \lambda)$, with

$$z_1(\beta, \lambda) \leq z_1(\beta, \lambda_1) < 1 < z_2(\beta, \lambda_2) \leq z_2(\beta, \lambda).$$

By continuity of functions under consideration, for any $\lambda \leq \lambda_1$, $\beta z_2(\beta, \lambda) \rightarrow z_2(1, \lambda)$ as $\beta \uparrow 1$, so one can choose $\beta_1 \in [\widehat{\beta}_1, 1)$ such that

$$\inf_{\lambda \in (0, \lambda_1]} \inf_{\beta \in [\beta_1, 1]} \beta z_2(\beta, \lambda) > 1.$$

Then, again for all $\lambda \in (0, \lambda_1]$ and all $\beta \in [\beta_1, 1]$,

$$j_1(z_1) - j_2(z_1) < 0 < j_1(z_2) - j_2(z_2), \quad (9)$$

with $z_i = z_i(\beta, \lambda)$, for $i = 1, 2$.

Now we fix $\beta \in [\beta_1, 1)$ and, for given $0 < \lambda_0 < \lambda_1$, let

$$D_0 = \frac{2}{\inf_{\lambda_0 \leq \lambda \leq \lambda_1} (j_2(z_1) - j_1(z_1))} < \infty. \quad (10)$$

Then, for any $D \geq D_0$ and for any $\lambda \in [\lambda_0, \lambda_1]$, we have $b_1(z_1) < 0$ and $b_1(z_2) > 0$ (the latter inequality always holds).

Take any $D \geq D_0$ and let $t_1 < t_2$ be the roots to equation $b_1(z) = 0$ (one can easily show that the latter equation has exactly two roots). Then, clearly, $0 < t_1 < z_1 < t_2 < z_2$.

Further, we may choose C such that all roots of equation $r(z) = 0$ lie in the interval (z_1, t_2) . Indeed, for $z > z_2 \geq 1$ and $\lambda \leq \lambda_1$,

$$\begin{aligned} r(z) &= \lambda - j(z) - Cz(1 - j(z)) - CDz(j_1(z) - j_2(z)) \\ &\leq \lambda_1 - C(1 - e^{-1}) \leq -1 \end{aligned}$$

if

$$C \geq C_1 := \frac{\lambda_1 + 1}{1 - e^{-1}}. \quad (11)$$

Further, for $z \leq t_1 \leq 1$ and $\lambda \geq \lambda_0$,

$$\begin{aligned} r(z) &> \lambda - j(z) - Cz(1 - j(z)) \\ &\geq \lambda - j(t_1) - Ct_1 \\ &\geq \lambda_0 - (1 + C)t_1 \end{aligned}$$

since $j(t_1) \leq t_1 e^{-t_1} \leq t_1$. The value of t_1 is decreasing to 0 as D tends to infinity. Therefore, one can choose $D_1 = D_1(C)$ such that $(1 + C)t_1 \leq \lambda_0/2$ for any $D \geq D_1$, and then let $D(C) = \max(D_0, D_1)$.

In more detail, since $\sup_{t_0 \leq t \leq 1} \frac{1-j(t)}{j_2(t)-j_1(t)} < \infty$ for any $0 < t_0 < 1$ and since $\lim_{t \downarrow 0} \frac{1-j(t)}{j_2(t)-j_1(t)} = \infty$, we may choose $D_1 = D_1(C)$ so large that

$$t(D_1) = \max\{t \in (0, 1) : \frac{1-j(t)}{j_2(t)-j_1(t)} \geq D_1\}$$

satisfies inequality $t(D_1) \leq \frac{\lambda_0}{2(C+1)}$. Therefore, for any $D \geq D_1$, we have $t_1 < t(D_1) \leq \frac{\lambda_0}{2(C+1)}$.

It is left to comment that $r(z) > 0$ for $z \in (t_1, z_1]$ since $a(z) \geq 0$ and $b(z) < 0$, and that $r(z) < 0$ for $z \in [t_2, z_2]$ since $a(z) < 0$ and $b(z) \geq 0$.

The proof of the lemma is complete.

Now we apply the lemma to the proof of Theorem 1 as follows.

Step 1. We introduce fluid limits for the Markov chain under consideration. Let Markov Chain $(N_k^{m_0}, S_k^{s_0})$ start from initial values $N_0 = m_0, S_0 = s_0$ and assume that $v := m_0 + s_0 \rightarrow \infty$ and that $m_0/v \rightarrow x, s_0/v \rightarrow y$ where $x + y = 1$. Consider a continuous time Markov process $(N^n(t), S^s(t))$ where

$$N^n(t) = N_{[tv]}^n, \quad S^s(t) = S_{[tv]}^s$$

and $[z]$ is an integer part of number z . Then we consider a family of weak limits of these processes, as $v \rightarrow \infty$. By following the standard scheme (see, e.g., [3], [1]), one can easily show that each such a limit, say $(\tilde{N}(t), \tilde{S}(t))$ is a Lipschitz function with continuous derivatives, and the derivatives are the functions $a(z)$ and $b(z)$ that were introduced earlier.

To see that the derivatives are $a(z)$ and $b(z)$ indeed, we may find the one-step drift. We have

$$j(m, s) := \mathbf{E}(J_n \mid N_n = m, S_n = s) = \frac{m\beta}{2s} \left(1 - \frac{\beta}{s}\right)^{m-1} + \frac{m}{2s} \left(1 - \frac{1}{s}\right)^{m-1}.$$

Then

$$a(m, s) := \mathbf{E}(N_{n+1} - N_n \mid N_n = m, S_n = s) = \lambda - j(m, s).$$

In the conditions of Theorem 1, and for $s > CD$,

$$b(m, s) := \mathbf{E}(S_{n+1} - S_n \mid N_n = m, S_n = ys) = C(1 - j(m, s)) + CD \left(\frac{m\beta}{2s} \left(1 - \frac{\beta}{s}\right)^{m-1} - \frac{m}{2s} \left(1 - \frac{1}{s}\right)^{m-1} \right).$$

Then, as $m + s \rightarrow \infty, m/s \rightarrow z$,

$$j(m, s) \rightarrow j(z), \quad a(m, s) \rightarrow a(z), \quad b(m, s) \rightarrow b(z).$$

Step 2. We show next that any fluid limit $(\tilde{N}(t), \tilde{S}(t))$ is *stable* in the following sense: for any $\varepsilon \in (0, 1)$, there exists finite time t_ε such that $\tilde{N}(t_\varepsilon) + \tilde{S}(t_\varepsilon) \leq 1 - \varepsilon$.

Then, by the general theory (see e.g. [1]), the positive recurrence of the underlying Markov chains follows.

Below we introduce a positive function $R(z)$ with $R(0) = 1$ and show that the pair $(\arctan(z), R(z))$, $0 \leq z \leq \infty$ is a smooth function whose graph splits the positive quadrant into two domains (where one of the domains is a convex compact neighbourhood of the origin) with vector $(a(z), b(z))$ being directed into the compact domain, and its normal component to the line is not smaller than a certain positive value.

The proof follows a number of routine steps, therefore we provide a sketch of the proof only.

First we construct a piecewise-linear function. Then we make it smooth around points where the line changes its direction.

Then, for any constant $c > 0$, we draw a line $(\arctan(z), cR(z))$, and introduce a test function $L(x, y)$ by letting $L(x, y) = c$ if the point (x, y) belongs to the line $(\arctan(z), cR(z))$.

We now introduce a piecewise linear function. We start from $(x_0, 0)$ with $x_0 = 1$ and draw a line L at an angle from the abscissa which is smaller than angle $\varphi_0 = \arctan(C/\lambda)$ (which is the direction of the derivative of the fluid limit at this point) by an angle α with tangent $\varepsilon \ll 1$. We assume that the ratio $K := (\pi/2 - \varphi_0)/\alpha$ is an integer.

Now we find $0 < z_1 < \dots < z_N$ such that $b(z_i)/a(z_i) = \tan(\varphi_0 + i\alpha)$ and draw the lines l_i that start from the origin in the direction $\arctan(z_i)$. Let (x_1, z_1x_1) be the point of the intersection of the lines L and l_1 . Starting from this point, we continue L in an angle φ_0 , until it crosses the line l_2 at point (x_2, z_2x_2) . Then we continue L under angle $\varphi_0 + \alpha$ until it intersects l_3 , and so on. When L meets l_N (at point x_N, z_Nx_N), it continues under the angle $\pi/2 - \alpha$ until the intersection with the line $y = x$ (which starts from the origin under angle $\pi/4$). Assume the intersection point is $(x^{(1)}, x^{(1)})$.

Now we start from $(0, y_0)$ with $y_0 = 1$ and draw another piecewise linear line in the upper half-quarter of the positive quadrant using similar arguments. Assume it intersects the line $y = x$ at point $(x^{(2)}, x^{(2)})$. If we start instead from $(0, \frac{x^{(1)}y_0}{x^{(2)}})$, then the two lines meet at point $(x^{(1)}, x^{(1)})$. The two lines divide the positive quadrant into two domains, and the compact domain is convex. The combined line is piecewise linear. We can make it "smooth" by local changes in the turn points. This completes a construction of the line $L(x, y) = 1$.

All other lines $L(x, y) = c$ are obtained by the scaling. Then, for c large enough, the line $L(x, y) = c$ is rectifiable. Then, using routine calculations, one can show that, at each point of the line, the drift vector is directed into the compact domain, and its normal projection is always positive (which means that its minimal value is still positive, due to continuity and boundedness of the line). This completes the proof of the theorem.

4 Conjectures

Here we introduce two more classes of transmission protocols and conjecture corresponding stability results.

Let \mathcal{H} be a class of functions $h : [1, \infty) \rightarrow [0, \infty)$ such that $h(1) = 0$, $h(x) \uparrow \infty$ is non-decreasing in x , $x - h(x) \uparrow \infty$ is non-decreasing in x , and $h(x)/x \rightarrow 0$, as $x \rightarrow \infty$. With each $h \in \mathcal{H}$,

we associate a class \mathcal{E}_h of positive functions $\varepsilon_h : [1, \infty) \rightarrow (0, 1/2]$, such that $\varepsilon_h(x) \rightarrow 0$ and $h(x)\varepsilon^2(x) \rightarrow \infty$, as $x \rightarrow \infty$.

The two other classes of algorithms differ from the first class in the following.

Algorithms from the second class \mathcal{A}_2 differ from those from the class \mathcal{A}_1 only in a way the S 's are updated: the constant CD is replaced by a function $h \in \mathcal{H}$. More precisely, the algorithms are determined by β , C , $h(x)$, $\{J_n\}$ and $\{I_n\}$. Given S_n , we again let

$$p_n = \begin{cases} \beta/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1, \end{cases}$$

but now define S_{n+1} by

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + h(S_n) & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - h(S_n), 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

For this algorithm, we use notation $A_2(C, h, \beta) \in \mathcal{A}_2$.

We modify further the class 2 algorithms by replacing β by $1 - \varepsilon_h$, this will form the third class \mathcal{A}_3 . More precisely, the algorithms are determined by C , h , ε_h , $\{J_n\}$ and $\{I_n\}$. Given S_n , we now let

$$p_n = \begin{cases} (1 - \varepsilon_h(S_n))/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1, \end{cases}$$

and then define S_{n+1} as for class 2 algorithms:

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + h(S_n) & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - h(S_n), 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

For this algorithm, we use notation $A_3(C, h, 1 - \varepsilon_h) \in \mathcal{A}_3$.

We can see that again, with any algorithm from the classes \mathcal{A}_2 or \mathcal{A}_3 , a sequence $\{(N_n, S_n)\}$ forms a time-homogeneous Markov chain.

We believe that the following two statement should be true.

Conjecture 1. Let $0 < \lambda_0 < e^{-1}$ be any number. There exists $C > 0$ and $\beta_2 \in (0, 1)$ such that, with any $\beta \in (\beta_2, 1)$ and any function $h \in \mathcal{H}$, algorithm $A_2(C, h, \beta)$ stabilizes the system, for any input rate $\lambda < \lambda_0$.

If, on the contrary, either $\beta < \beta_2$ or $\lambda > \lambda_0$, then the algorithm $A_2(C, h, \beta)$ is unstable in the system with input rate λ , for any $h \in \mathcal{H}$.

Conjecture 2. Any algorithm $A_3(C, h, 1 - \varepsilon_h)$ from the third class stabilizes the system, for any input rate $\lambda < e^{-1}$.

Remark 2. The conjectures may hold for a broader classes of algorithms if one assumes that, in the recursion for S_n , function h is replaced by two functions, h_1 in the second line and h_2 in the third line.

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